Abstract—The initial alignment is a key technology for strapdown inertial navigation system (SINS), because the performance of the SINS is largely decided by the accuracy and rapidity of the alignment process. In this paper, a novel alignment method is developed by combining the quaternion Kalman filter with optimization-based alignment idea. The process model and measurement model of the proposed alignment method are constructed based on the properties of quaternion. Since the correlation measurement noise in the measurement formula, the adaptive filter of the quaternion Kalman filter combined with innovation-based adaptive estimation is developed, which calculate the filter gain matrix by the innovation covariance online. The simulation and experimental results demonstrate that the proposed alignment method has more accurate alignment results compared with the existing alignment methods.

Keywords—quaternion Kalman filter; adaptive estimation algorithm; fading memory

I. INTRODUCTION

Strapdown inertial navigation initial alignment is one of the key technologies in the SINS research, it is the focus of research at home and abroad. SINS initial alignment is generally divided into two stages: coarse and fine alignment. Most of the coarse alignment is done by the analytic method, the fine alignment stage generally uses state estimation method, such as Kalman filter. The current fine alignment methods based on Kalman filtering rely on the coarse alignment stage to provide a known initial attitude [1]. The attitude representation method is the basis for studying the problem of attitude estimation. There are four parameters of the quaternion representation method and three parameters of the rotation vector. Different representations have their particularities. However, one of the most prominent features of this is that any three-parameter attitude representation method can’t be globally non-singular, and the four-parameter quaternion representation method has a global non-singular advantage. So the quaternion becomes the most widely used representation of gesture [2].

Quaternion has been used for a long history in mathematics. Optimal algorithms have been developed over the last four decades following two main approaches; namely, the classical least-squares approach and the Kalman filtering approach. The first approach was the Wåhba’s problem [3], which is a constrained least-squares minimization problem for finding the attitude matrix. Davenport formulated and solved Wåhba’s problem in terms of the attitude quaternion [4] in 1971. The developed algorithm is known in the literature as the q-method. The benefits of quaternion are conveniently illustrated in the spacecraft orientation tracking where the aim is to find the mapping between the coordinate system on a reference-frame and a local frame on the spacecraft body frame. Numerous algorithms were developed in order to provide the basic q-method with other features like the ability of sequentially estimating time-varying attitude [5, 6] and of estimating parameters other than attitude [6, 7].

On the other hand, the Kalman filter method is designed to produce a continuous quaternion estimate as the minimum variance and allows the estimation of parameters other than the posture in a direct way. However, the Kalman filter is not designed to preserve the weaving applied to the estimated state variables. It is difficult to maintain the unit norm attribute of the quaternion by various means. We can use the extended Kalman filter to calculate the attitude, but when the system is non-linear, EKF filter stability and accuracy are poor.

The linear form of the quaternion vector observation equation is obtained in the literature [8], and deduced the corresponding quaternion Kalman filter. The quaternion Kalman filter algorithm was tested in literature [9], the result showed that the performance of the quaternion Kalman filter was better than the EKF. Based on the quaternion Kalman filter, the online adaptive estimation of gyro noise covariance Q was designed in the literature [8], but it didn’t propose an adaptive algorithm to calculate the noise covariance R of the accelerometer online aiming at the quaternion Kalman filter. Acceleration data is sensitive to mechanical vibration, and the statistical characteristics of the acceleration noise are measured in the flight state; at the same time, the statistical characteristics of the acceleration noise will vary with the aircraft body and the external environment. Therefore, it is necessary to estimate the covariance matrix of the acceleration noise using the adaptive quaternion Kalman filter. Considering the accelerometer noise covariance R is unknown or changing, the author refers to the method of estimating gyro noise covariance Q by adaptive algorithm in the literature [8], designs the innovation-based adaptive estimation algorithm to estimate the acceleration noise covariance R online, and carries out the simulation.

The contents are organized as follows: In section 2 we describe the novel quaternion alignment based on the Kalman filter. In section 3 an improved algorithm based on the quaternion Kalman filter is developed. Simulation results are reported in Section 4. Finally, in the last section, we present the conclusions derived from this work.
II. ATTITUDE ALIGNMENT BASED ON THE QUATERNION METHOD

It is well known that the acceleration measured by inertial measurement unit (IMU) axes (b-frame) is caused by the natural inertial acceleration of the vehicle and apparently caused by the motion of b frame relative to the i frame. The former element can be compensated by external sensors, and acceleration in the b-frame can be calculated by the acceleration measurements and the gyroscope measurements. That is how the true inertial acceleration is compensated. Taking advantages of the known position, the true gravity in the n0-frame which is named the reference vector can be obtained accurately.

A. Attitude Determination

Denote by N the local level navigation frame, by B the SINS body frame, by I the inertia non-rotating frame, by E the Earth frame. The navigation (attitude, velocity and position) rate equations in the N-frame are respectively known as [8, 10,]

$$\dot{C}_n^b = C_n^b \omega_b^n \times \text{ (1) }$$

$$\dot{V} = C_n^b f^b - (2\omega_b^n + \alpha_b^n) \times V^n + g^n \text{ (2) }$$

where, $C_n^b$ denotes the attitude matrix from the body frame to the navigation frame, $\omega_b^n$ the body angular rate measured by gyroscopes in the body frame, $V^n$ the velocity relative to the Earth (also called ground velocity), $f^b$ the specific force measured by accelerometers in the body frame, $\alpha_b^n$ the Earth rotation rate with respect to the inertial frame, $\alpha_b^n$ the angular rate of the navigation frame with respect to the Earth frame, $\omega_b^n = \omega_b^e - C_b^e \omega_b^e$ is the body angular rate with respect to the navigation frame, and $g^n$ is the gravity vector. The $(\times)$ is the matrix form of a cross-product.

Equation (1) is the traditional attitude update equation. Nowadays, the attitude update is separated using the DCM product chain rule as [11]

$$C_{n(t+\Delta t)}^{n(t)} = C_{b(t+\Delta t)}^{n(t+\Delta t)} C_{b(t)}^{n(t)} C_{b(t)}^{b(t)} \text{ (3) }$$

If this rule is applied in each update cycle, the attitude update equation will be:

$$C_{n(t)}^{b(t)} = C_{n(0)}^{b(t)} C_{b(t)}^{b(0)} C_{b(0)}^{n(0)} \text{ (4) }$$

The update equations for $C_{n(t)}^{b(t)}$ and $C_{b(t)}^{b(t)}$ are

$$C_{n(t)}^{n(t)} = C_{n(0)}^{n(t)} C_{b(t)}^{b(0)} \text{ (5) }$$

$$C_{b(t)}^{b(t)} = C_{b(0)}^{b(t)} C_{b(0)}^{n(0)} \text{ (6) }$$

Substituting (4) into (2) yields

$$\dot{V} = C_{n(0)}^{n(t)} C_{n(t)}^{b(t)} f^b - (2\omega_b^n + \alpha_b^n) \times V^n + g^n \text{ (7) }$$

Multiplying $C_{n(t)}^{n(0)}$ on both sides, we can get

$$C_{n(t)}^{n(0)} \alpha_t = \beta_t \text{ (9) }$$

$$\alpha_t = \int_0^t C_{n(t)}^{n(0)} f^b dt \text{ (10) }$$

$$\beta_t = C_{n(t)}^{n(t)} V^n - V^0 + \int_0^t C_{n(t)}^{n(0)} \omega_b^n \times V^n dt - \int_0^t C_{n(t)}^{n(0)} g^n dt \text{ (11) }$$

As shown in [12], equation (8) can be solved by the optimization-based method using the unit attitude quaternion parameter. Specifically, the four-element unit quaternion $q = [s \quad \eta]^T$, where $s$ is the scalar part and $\eta$ is the vector part, is used to encode the initial body attitude matrix $C_b^e(0)$.

$$C_b^e(t) = (s^2 - \eta^2 \eta^T + 2\eta \eta^T - 2s(\eta^T)) \text{ (12) }$$

Define the quaternion multiplication matrices by

$$[q] = \begin{bmatrix} s \quad \eta \end{bmatrix} \begin{bmatrix} s \quad -\eta \end{bmatrix} = \begin{bmatrix} s \quad -\eta \end{bmatrix} \text{ (13) }$$

Then (11) is equivalent to

$$[\dot{q}] = \left[ \begin{bmatrix} \beta(t) \end{bmatrix} \cdot [\dot{q}] \right] q = 0, \text{ and the determination of the attitude quaternion can be posed as a constrained optimization [10].}$$

$$\min_q q^T K q, \text{ subject to } q^T q = 1$$

Where the real symmetric matrix

$$K = \left[ \begin{bmatrix} \beta(t) \end{bmatrix} \cdot [\dot{q}] \right] \left( [\beta(t)] \cdot [\dot{q}] \right) \text{ (14) }$$

It can be proved that the optimal quaternion is exactly the normalized eigenvector of $K$ belonging to the smallest eigenvalue [12].

B. Quaternion Kalman Filter

According to the reference [13], the complete process of the initial attitude quaternion Kalman filter is as follows:

$$q_{k+1} = q_k \text{ (15) }$$

$$0 = H_{k+1} q_{k+1} - \frac{1}{2} \Xi(q_{k+1}) \delta b_{k+1} \text{ (16) }$$

$$H_k = \left[ -\frac{1}{2} (\alpha_k + \beta_k) \times \frac{1}{2} (\alpha_k - \beta_k) \right]$$

$$\Xi(q_{k+1}) = \left[ \eta_{k+1} \times + sI_3 \right] \text{ (18) }$$

$$\dot{M}_{k+1} = \dot{q}_{k+1}^T \hat{q}_{k+1} + p_k^T \text{ (19) }$$

$$B_{k+1} = H (\alpha_{k+1}) \text{ (20) }$$

$$p_{k+1} = -R_{k+1} \left[ \tr(\dot{M}_{k+1}) I_k - \dot{M}_{k+1} H_{k+1} \dot{M}_{k+1} + \dot{p}_{k+1} \right] \text{ (21) }$$

$$S_{k+1} = H_{k+1}^T p_{k+1}^T H_{k+1} + p_{k+1}^T \text{ (22) }$$

$$K_{k+1} = p_{k+1}^T H_{k+1}^T S_{k+1}^{-1} \text{ (23) }$$

$$\dot{q}_{k+1} = \left( I_4 - K_{k+1} H_{k+1} \right) \dot{q}_{k+1} + K_{k+1} p_{k+1} \text{ (24) }$$

$$p_{k+1} = (I_4 - K_{k+1} H_{k+1}) p_{k+1} \text{ (25) }$$

$$q_{k+1} = \frac{\hat{q}_{k+1}}{\|\hat{q}_{k+1}\|} \text{ (26) }$$
Let $\hat{Q}_{k+1}$ denote the expected value of $q_{k+1}$, the explicit dependence on $q_{k+1}$ is expressed. Let $P_{k+1}$ and $R_{k+1}$ denote the covariance matrices of $q_{k+1}$ and $v_{k+1}$ respectively.

III. THE IMPROVED QUATERNION KALMAN FILTER

In this section, we introduce the adaptive filter of the quaternion Kalman filter combined with innovation-based adaptive estimation and analyze the principle of this improved algorithm.

A. The adaptive quaternion kalman filter based on innovation

The measurement residuals process at $t_{k+1}$ is given by

$$v_{k+1/k} = -H_{k+1}\hat{q}_{k+1/k}$$

(27)

$$A_k = d_k v_{k+1/k} v_{k+1/k}^T + (1 - d_k) A_{k-1}$$

(28)

$$d_k = 1 - b - (1 - b^2)$$

(29)

$$C_k = \frac{1}{4} [\text{tr}(\hat{M}_{k+1} - \hat{M}_{k+1} - B_v \hat{M}_{k+1} B_v^T)]$$

(30)

$$W_k = H_{k+1} P_{k+1/k} H_{k+1}^T$$

(31)

$$\rho_{k+1} = \frac{\text{tr}[(m A_k - W_k) C_k]}{\text{tr}(C_k C_k^T)}$$

(32)

$$\rho_{k+1} = \begin{cases} \rho_{k+1} \geq 0 & \\
0 & \rho_{k+1} < 0 \end{cases}$$

(33)

$$R_{k+1} = \rho_{k+1} I_3$$

(34)

Perform (27) ~ (34) adaptive process before (21) and obtain the $R_{k+1}$. Replacing the $R_{k+1}$ in equation (21) and then continue to calculate the quaternion Kalman filter.

B. The AdaptiveAlgorithmPrinciple Analysis

According to (16) we can get

$$\hat{S}_{k+1/k} = \frac{1}{N} v_{k+1/k} v_{k+1/k}^T$$

(35)

Where, $v_{k+1/k}$ is the quaternion-dependent noise, $\hat{S}_{k+1/k}$ is the covariance matrix of the $v_{k+1/k}$. It has been proved that $\hat{S}_{k+1/k}$ is the maximum likelihood optimal estimation of $S_{k+1/k}$ which used the maximum likelihood method in reference [14]. According to (21) and (22), we know that $S_{k+1/k}$ is the function of $\rho_{k+1}$ namely $S_{k+1/k}(\rho_{k+1})$. In order to get the right $\rho$, we should let the difference between sampling covariance matrix and the value of forecast $S_{k+1/k}$ as small as possible, that is to say solve the following minimization problem:

$$\min_{\rho_{k+1} > 0} \left\{ J(\rho_{k+1}) = \|\hat{S}_{k+1/k} - S_{k+1/k}(\rho_{k+1})\|^2 \right\}$$

(36)

Where the $\|\hat{S}_{k+1/k} - S_{k+1/k}(\rho_{k+1})\|$ means the Frobenius norm that is $\|\hat{S}_{k+1/k} - S_{k+1/k}(\rho_{k+1})\| = \text{tr}(\hat{S}_{k+1/k} - S_{k+1/k}(\rho_{k+1})) \cdot (\hat{S}_{k+1/k} - S_{k+1/k}(\rho_{k+1}))^T$. The value of $\rho_{k+1}$ which is computed from (36) represents the optimal estimate of the process noise levels. Substituting the equation (21) into (22), we can get

$$S_{k+1/k}(\rho_{k+1}) = H_{k+1} P_{k+1/k} H_{k+1}^T + \frac{1}{4} \rho_{k+1} [\text{tr}(\hat{M}_{k+1} - \hat{M}_{k+1} - B_v \hat{M}_{k+1} B_v^T)]$$

(37)

$$W_k = H_{k+1} P_{k+1/k} H_{k+1}^T$$

(38)

$$C_k = \frac{1}{4} \text{tr}(\hat{M}_{k+1} - \hat{M}_{k+1} - B_v \hat{M}_{k+1} B_v^T)$$

(39)

Referring to the method of appendix B in [7], minimize (36) and we can get

$$\rho_{k+1} = \frac{\text{tr}[(m A_k - W_k) C_k]}{\text{tr}(C_k C_k^T)}$$

(40)

Where, $m$ is used to adjust the adaptive effect of scaling factor.

If $\rho_{k+1}$ is positive, let $\rho_{k+1}$ update the $\rho_{k+1}$, then calculate $\rho_{k+1}$, if $\rho_{k+1}$ is negative. Let $\rho_{k+1}$ be 0. The reason of $\rho_{k+1}$ being 0 when $\rho_{k+1}$ is negative is as follows: We can interpret the matrix $C_k$ in (39) as the residual covariance matrix that is predicted by the filter as if there were no process noise in the system. On the other hand, the matrix $v_{k+1/k} v_{k+1/k}^T$ is an approximate value for the residual covariance as computed from the actual measurements. If $C_k$ will be greater than $v_{k+1/k} v_{k+1/k}^T$, this means that the filter is too conservative and should lower its level of process noise in order to fit the actual residual level. However, we rule out such a value in order to ensure that the time propagation stage only increases the estimation error covariance, as should be the case in a well KF.

Accelerometer noise will change because of the change of the vehicle body and the external environment, the acceleration noise covariance $\rho_{k+1}$ also changes. So for the time-varying covariance, valuation should gradually forget the out-of-date information. We adopt the fading memory of square method and get $\hat{S}_{k+1/k}$. The $A_k$ is designed using the fading memory of square method, $b$ is forgetting factor $0 < b < 1$. Fading memory method can slowly reduce the weight of the old data, it make adaptive estimation more sensitive to accelerometer noise change reaction and reduce the storage space utilization. In this paper, we use the fading memory method to measure the covariance. Figure 1 shows
the structure block diagram of the adaptive quaternion Kalman filter based on the innovation.

![Block Diagram](image)

**Fig.1. The chart of the adaptive quaternion Kalman filter based on innovation**

IV. SIMULATION RESULTS AND ANALYSIS

To test and verify the effectiveness of the proposed alignment algorithm, we carry out simulations in this section. The parameters of the simulation are set as follows:

The underwater SINS location of the simulation is set as follows: north latitude is 40°, east longitude is 118°. And the vehicle is rocked by the surf and undercurrent. The pitch, roll and heading resulting from the vehicle rocking are changed periodically and can be described as follows:

\[
\theta = 7 \cos(\frac{2\pi}{5}t + \frac{\pi}{4}) \]  
\[
\gamma = 10 \cos(\frac{2\pi}{6}t + \frac{\pi}{7}) \]  
\[
\psi = 30 + 5 \cos(\frac{2\pi}{7}t + \frac{\pi}{3}) \]

The IMU errors are set as follows: the gyro constant drift: 0.01\(^\circ\)/h, the gyro random noise: 0.001\(^\circ\)/h, the accelerometer bias: 1\times10^{-4}\text{g}, and the accelerometer measurement noise: 1\times10^{-4}\text{g}. The SINS sampling rate is 100 Hz. The incremental update interval \(T=0.02\). The simulation running time is 300 seconds. In the absence of such initial estimate, choose the zero attitude quaternion; that is \(\hat{q}_0=[0 0 0 1]^T\), the initial estimation error covariance matrix \(\Gamma=0.02\). The measurement innovation covariance \(\Gamma_{m}=51\), The forgetting factor \(\beta\) is 0.01.

The figure 2 shows that when the acceleration noise covariance is unknown, the error range of attitude estimation is 2 degree at the time from 0 to 150s, the error range of attitude estimation is 0.1 degree at the time from 150s to 300s, the reason is follows: we didn’t use the adaptive quaternion kalman filter based on innovation, when the three-axis accelerometer noise covariance matrix was unknown, the quaternion kalman filter algorithm will introduce the noise of accelerometer into the state variables, which resulted in the attitude estimation angle containing a large number of high-frequency noise and reduced the attitude estimation accuracy. From the 150s later, we began to use the improved algorithm. The adaptive algorithm based on the innovation would adjust the three-axis accelerometer noise covariance matrix parameter, attenuated the accelerometer noise into the estimated attitude. It is not necessary to know the value of the covariance matrix of the three-axis accelerometer and the attitude can be well estimated. Thus, the improved algorithm can reduce the dependence of the filter on the model parameters, and even if the variance of the accelerometer noise is unknown, the attitude estimation can be well performed. From figure 3 we can know that when the acceleration noise covariance is known and changing, the performance of the quaternion kalman filter is worse than the improved algorithm. Because the adaptive algorithm can on-line estimate the
accelerometer data covariance matrix according to the innovation, the adaptive algorithm can effectively reduce the influence of constantly unknown changing noise on attitude estimation and has good robustness.

V. CONCLUSION

Initial alignment by optimal attitude determination has been proven to be an attractive alternative for the traditional initial alignment methods. On the idea of optimization-based alignment, this paper develops a novel alignment method by combining the quaternion Kalman filter with optimization-based alignment method. In this method, the linear process formula and measurement formula are constructed basing on the properties of quaternion. For the correlation measurement noise problem, the adaptive quaternion Kalman filter based on innovation adaptive estimation derived by us is applied. The simulation results demonstrate that effectiveness of the proposed initial alignment method is superior, and adaptive quaternion Kalman filter solves the initial alignment attitude estimation problem is more accurate and stable than the existing algorithm.

ACKNOWLEDGMENT

The authors would like to thank reviewers for their valuable comments, which significantly improved the presentation of this paper.

REFERENCES