A Prediction Method of Spacecraft Telemetry Parameter Based On Chaos Theory

Li Lei, Gao Yongming, Wu Zhihuan, Ma Kaihang
Department of Graduate Management
Equipment Academy
Beijing, China
e-mail: lilei_0617@hotmail.com

Abstract—Spacecraft state prediction is an important method to solve the related problems of spacecraft health management, fault diagnosis and so on. Chaos theory is widely used in electric power and machinery industries. It can describe the whole system with some parameters. It is proved that spacecraft telemetry parameters are chaotic by calculating the chaotic characteristics. Aiming at the problems of complicated spacecraft system, complicated parameters and difficult to establish accurate mathematical model, a prediction method based on phase space reconstruction is proposed. The adaptive feature extraction of spacecraft telemetry data is realized by phase space reconstruction, and the prediction of telemetry data is realized by RBF neural network and Volterra filter. The experimental results show that the proposed method has better prediction performance.

Keywords—chaos; phase space reconstruction; RBF neural network; Volterra filter; parameters prediction component

I. INTRODUCTION

Chaos is an external, complex, seemingly random motion in deterministic systems due to intrinsic randomness. Chaos is not disorder, more like a non-cyclical order. In 1903, chaos was first observed by Poincare H in his study of three-body problem. He put the dynamics system and the topology organic combination, proposed that three-body problem solution was random in a certain rang. Since then, the study of chaos theory has played a big role in dynamical system, even in nonlinear science[1-5].

Spacecraft is a large-scale complex system with combination of advanced technology in many areas, it has an important impact on national economy, science and technology and other activities. The special space environment where the spacecraft works makes the faults situation very complex and difficult to diagnose and repair. Therefore, the study of spacecraft faults has important social and economic benefits and engineering value. During the operation of the spacecraft, there will be many kinds of telemetry data, which can directly or indirectly reflect the state of the spacecraft components. The fault of spacecraft often lead to the anomaly of these parameters, it is very important for the early detection of spacecraft faults by analyzing the trend of spacecraft telemetry parameters and the variation law.

The unpredictability of chaotic system means that it is impossible to predict quantitatively in the long term, but the determinacy of chaos could be used to make accurate and quantitative prediction in the short term. Based on the theory of embedded phase space, this paper uses a series of methods of chaos theory to calculate the chaotic characteristics of spacecraft telemetry data with satellite flywheel current data as an example. Spacecraft telemetry parameters prediction is made by RFB neural network and Volterra filter, the simulation results show that the proposed method has a good prediction performance and is a simple and easy method for short-term prediction of spacecraft telemetry parameters.

II. CHAOTIC CHARACTERISTICS ANALYSIS

In general, a time series is chaotic when it has a positive maximum Lyapunov exponent, a finite correlation dimension and a finite positive Kolmogorov entropy. In order to determine whether the telemetry parameters are chaotic, three feature quantities are computed respectively.

A. Phase Space Reconstruction

Takens theorem, proposed by Takens in 1981, shows that information of the original system could be recovered in the embedded space with a proper embedding dimension[6]. The Lyapunov exponent and other chaotic characteristics of the embedded phase space are the same as those of the original system, which means that the original laws of the system could be extracted and recovered from a batch of time series data of a certain component in the system. Further, Packard et al. Proposed two phase space reconstruction methods: derivative reconstruction method and coordinate delay reconstruction method. Coordinate delay method is often used to reconstruct phase space in practice.

For an univariate time series \( \{x(i)\} \), an m-dimensional phase space is constructed with the time lag parameter \( \tau \) and the embedding dimension m:

\[
X_i = [x_i, x_{i+\tau}, \cdots, x_{i+(m-1)\tau}] \quad (1)
\]

where: \( i = 1, 2, \cdots, L \); \( L = N -(m-1)\tau \). The reconstructed phase space matrix is:
\[
\begin{align*}
\mathbf{X}_1 &= \left[ \mathbf{x}_1, \mathbf{x}_1 + \tau, \cdots, \mathbf{x}_1 + (m - 1)\tau \right] \\
\mathbf{X}_2 &= \left[ \mathbf{x}_2, \mathbf{x}_2 + \tau, \cdots, \mathbf{x}_2 + (m - 1)\tau \right] \\
&\vdots \\
\mathbf{X}_n &= \left[ \mathbf{x}_n, \mathbf{x}_n + \tau, \cdots, \mathbf{x}_n + (m - 1)\tau \right]
\end{align*}
\] 

(2)

Takens theorem determines the basis of the selection of embedded dimensions, but the actual system dynamics dimension is usually unknown. The key issue of phase space reconstruction is how to select the appropriate delay \( \tau \) and embedding dimension \( m \) when the length of the data sequence is limited. There are two main ways to determine the delay and the embedding dimension. One is to select \( \tau \) and \( m \) independently which means \( \tau \) and \( m \) are independent of each other, such as autocorrelation function method, mutual information method, G-P algorithm or pseudo-nearest neighbor method, etc. The other one is to calculate \( \tau \) and \( m \) at the same time, which means \( \tau \) and \( m \) are interdependent, such as embedded window method, C-C method\(^5\).

![Figure 1. C-C method](image)

The paper uses C-C method to compute \( \tau \) and \( m \), the computing result of one parameter is as shown in Fig.1. It could be seen from Fig.1 that the first local minimum point of curve \( \Delta S \) is 2 and the global minimum point of curve \( \Delta S \) is 6. Then \( m \) should be 4 since \( \tau_m = (m - 1) \tau \). There are 5 groups of parameters, and the results are shown in Table 1.

B. Largest Lyapunov Exponent

Rosenstein improved the Wolf method based on the orbit tracking method, and proposed a small data sets method for computing the largest Lyapunov exponent\(^6\). The method makes full use of all the available data, and obtains high precision. Small data sets method is fast, easy to implement. Specific steps are as follows:

1) For an univariate time series \( \{x(i)\} \), an \( m \)-dimensional phase space is constructed with the time lag parameter \( \tau \) and the embedding dimension \( m \):

\[
\mathbf{X}_i = \left[ \mathbf{x}_i, \mathbf{x}_i + \tau, \cdots, \mathbf{x}_i + (m - 1)\tau \right]
\]

(3)

where \( i = 1, 2, \cdots, L \); \( L = N - (m - 1)\tau \).

2) After reconstructing the dynamics, the algorithm locates the nearest neighbor of each point on the trajectory. The nearest neighbor \( \mathbf{X}_j \) is found by searching for the point that minimizes the distance to the particular reference point \( \mathbf{X}_i \).

\[
d_j(0) = \min \| \mathbf{X}_i - \mathbf{X}_j \| > p
\]

(4)

where \( \| \mathbf{X}_i - \mathbf{X}_j \| \) denotes the Euclidean norm, and \( p \) is the mean period of the time series;

3) For each point \( \mathbf{X}_i \), the distance from the \( i \)-th discrete time step of the nearest neighbor \( \mathbf{X}_j \) is calculated:

\[
d_j(i) = \| \mathbf{X}_i(j + i) - \mathbf{X}_j(j + i) \|
\]

(5)

Taking the logarithm of both sides of (6):

\[
\ln d_j(i) = \ln C_j + \lambda_i(i\Delta t)
\]

(7)

Equation (7) represents that curve \( i \sim \ln d_j(i) \) meets the linear relationship in a certain range. The largest Lyapunov exponent is easily and accurately calculated using a least-squares fit to the “average” line defined by

\[
y(i) = \frac{1}{q\Delta t} \sum_{j=1}^{q} \ln d_j(i)
\]

(8)

where \( q \) is number of non-zero of \( d_j(i) \).

5) Select a linear region of the curve \( i \sim y(i) \), and the slope of the straight line is the largest Lyapunov exponent \( \lambda_i \):

\[
\lambda_i = \frac{\sum_{i=1}^{q} y(i) - \tau \sum_{i=1}^{q} i}{\sum_{i=1}^{q} i}\quad i \neq 0
\]

(9)

\[
\Gamma = \frac{1}{n} \sum_{i=1}^{n} i
\]

\[
\mathbf{y} = \frac{1}{n} \sum_{i=1}^{n} y(i)
\]

Where \( n \) is the number of points satisfying the linear relation in the scale-free region.

C. Fractal Dimension and Kolmogorov Entropy

Zhao Guibing proposed a method of computing fractal dimension and Kolmogorov entropy from chaotic time series on the basis of G-P algorithm. Specific steps are as follows:

1) For an univariate time series \( \{x(i)\} \), an \( m \)-dimensional phase space is constructed with the time lag parameter \( \tau \) and the embedding dimension \( m \) as (3);

2) Calculating correlation integrals

\[
C(r) = \frac{1}{L^2} \sum_{i,j=1}^{L} \theta(r - \| X(i) - X(j) \|)
\]

(10)

where \( \theta(\cdot) \) is Heaviside unit function \( \theta(x) = \begin{cases} 0, & x \leq 0 \\ 1, & x > 0 \end{cases} \)

and \( \| X(i) - X(j) \| = \max_{0 \leq k \leq m-1} | x(i+k\tau) - x(j+k\tau) | \).
3) In the scale-free region of $r$, the log-log plot $\ln(r) \sim \ln(C(r))$ satisfies the linear relationship.

Let $x_i = \left[ \ln(r) \right]$ and $y_i = \left[ \ln(C(r)) \right]$, then

$$y_i = ax_i + b$$

where $a = D2$ and $K2 = \lim_{m \to \infty} \frac{\Delta b_i}{m}$.

4) Using the least squares to obtain the optimal estimate of $a, b$:

$$a = \frac{\sum_{i} \sum_{j} x_{ij} (y_{ij} - \bar{y}_i)}{\sum_{i} \sum_{j} x_{ij}^2}$$

$$b = \bar{y}_i - a \bar{x}_i$$

where $\bar{x}_i = \frac{1}{n_i} \sum_{j} x_{ij}$ and $\bar{y}_i = \frac{1}{n_i} \sum_{j} y_{ij}$.

Fig. 2 shows the log-log plots $\ln(r) \sim \ln(C(r))$ with different $m$. There are another 4 groups of parameters, and the results are shown in Table 1.

<table>
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<th>CHAR</th>
<th>PRM 1</th>
<th>PRM 2</th>
<th>PRM 3</th>
<th>PRM 4</th>
<th>PRM 5</th>
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<td>3</td>
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<td>2</td>
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<tr>
<td>$m$</td>
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<td>2</td>
<td>3</td>
<td>2</td>
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<td>$\lambda$</td>
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<td>0.1602</td>
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<td>6.2344</td>
<td>4.0441</td>
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</tr>
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</table>

All 5 groups of parameters have a positive maximum Lyapunov exponent, a finite correlation dimension and a finite positive Kolmogorov entropy, so spacecraft telemetry parameters are chaotic.

III. PREDICTION OF SPACECRAFT TELEMETRY PARAMETERS

In the previous section, the chaotic characteristics of the telemetry parameters are proved by calculating the chaotic features. Chaotic system is still a nonlinear system. RBF neural network and Volterra filter could fit nonlinear system well, this paper uses these two methods for telemetry parameter prediction.

A. RBF Neural Network

RBF neural network is based on radial basis function, the algorithm selects Gaussian function as the core function of RBF neural network[10]. Gaussian function has a monotonically decreasing property and has a good local characteristic: it is relatively significant only in a certain region near the central point, and its value approaches 0 gradually as the distance from the central point increases. Therefore, the radial basis function with local approximation capability is also called the local perceptual field neural network, which is widely used in practice. The general expression of radial basis function is:

$$h(x) = \Phi((x-c)^TE^{-1}(x-c))$$

Where $c$ is center vector of the function; $h$ is radial basis function; $E$ is the change matrix; $(x-c)^TE^{-1}(x-c)$ is the measure of the distance between the input vector $x$ and the center $c$ in the case where the matrix $E$ is determined. If $E$ is an Euclidean matrix, then $E = r^2I$, where $r$ is radius of radial basis function, then (12) could be simplified as:

$$h(x) = \Phi(\|x-c\|/r^2)$$

The general RBF neural network is a three-layer structure: the input layer, the hidden layer and the output layer. The input layer of RBF neural network mainly consists of input signal source nodes, and the hidden layer is composed of radial basis function. The spatial variation of input layer to hidden layer is performed by nonlinear radial basis function. The space transform from hidden layer to output layer is linear transformation, which means the output value of the output layer nodes are obtained by linearly weighting the output signal of the hidden layer.

B. Volterra Filter

Volterra series is a functional series which was proposed by the Italian mathematician Vito Volterra in 1880 as the promotion of Taylor series[11]. According to the Weierstrass theorem in function approximation theory, any continuous function defined in a closed interval could be arbitrarily approximated by a polynomial. It can be seen from the above section that the reconstructed dynamical system is equivalent to the motive force system in the topological sense when constructing the $m$-dimensional phase space with the appropriate time delay parameter $\tau$ and embedding dimension $m$. The state of the next time can be obtained from the current state of the system, so as to realize the prediction of the chaotic sequence. The essence of the prediction of chaotic time series is to reconstruct the model $F()$ of the system by obtaining the system state:

$$x(n + T) = F(X(n))$$

Where $T$ is the forward prediction step. Next the nonlinear predictive model $f$ will be constructed by the Volterra series to approach $F$. 

Let the input of the nonlinear discrete dynamical system be:
\[ \mathbf{x}(n) = [x(n), x(n-\tau), \ldots, x(n-(m-1)\tau)] \] (15)

Let output be \( y(n) = x(n+1) \). Then the nonlinear system function Volterra series expansion is
\[ x(n+1) = h_0 + \sum_{k=1}^{\infty} y_k(n) \] (16)

Where \( y_k(n) = \sum_{i_1, \ldots, i_k=0}^{m-1} h_k(i_1, \ldots, i_k) \prod_{j=1}^{k} x(n-i_j\tau) \), and \( h_k(i_1, \ldots, i_k) \) is called the k-order Volterra kernel, \( P \) is the Volterra filter order.

C. Simulation Results

Parameter 1 is chosen as an example, the first 1000 points of parameter 1 are training set and the after 500 points are test set. M-dimensional phase spaces are constructed with the time lag parameter \( \tau=2 \) and the embedding dimension \( m=4 \) for training set and test set. The phase spaces will be the input of RBF neural network and Volterra filter. RBF neural network and Volterra filter are used for training and prediction. RBF neural network and Volterra filter learn through the training set, and obtain the prediction models.

Figure 3. Prediction result of RBF
Figure 4. Absolute error of RBF
Figure 5. Relative error of RBF
Figure 6. Prediction result of Volterra filter
Figure 7. Absolute error of Volterra filter
Figure 8. Relative error of Volterra filter
The prediction are as shown as in Fig. 3~Fig. 8. Most of the prediction errors using the RBF neural network are less than 6%, but at some points they will increase to 1%. All of the prediction errors using Volterra filter are less than 6%. The prediction mean square error of RBF neural network and Volterra filter are respectively $4.0774 \times 10^{-4}$ and $3.7664 \times 10^{-4}$. Volterra filter performs better than RBF neural network.

IV. SUMMARY

In this paper, it is proved that spacecraft telemetry parameters are chaotic by calculating the chaotic characteristics. A prediction method based on chaos theory is proposed to predict spacecraft telemetry parameters. The method uses phase space reconstruction to extract the feature of telemetry parameter sequence, and then uses RBF neural network and Volterra filter to predict. The simulation results show that the two methods have good prediction effect, while the Volterra filter is better. The prediction method using chaos theory has the feature of feature extraction adaptive. For a telemetry parameter sequence, the features could be extracted by determining the delay and dimension of the reconstructed phase space, which is characterized by the nature of the time series itself. However, this method also has problems like the large amount of computation, and multiple iterations of the model.

REFERENCES